

Classification of Polar Equations

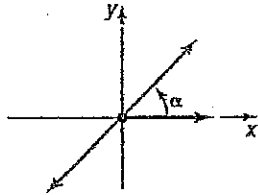
Lines

Description Line passing through the pole making an angle α with the polar axis

Rectangular equation $y = (\tan \alpha)x$

Polar equation $\theta = \alpha$

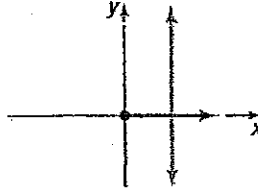
Typical graph



Vertical line

$x = a$

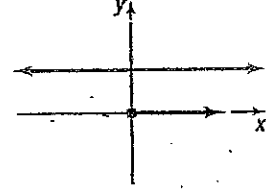
$r \cos \theta = a$



Horizontal line

$y = b$

$r \sin \theta = b$



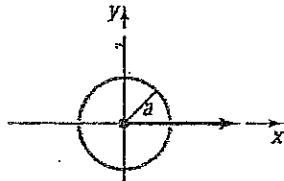
Circles

Description Center at the pole, radius a

Rectangular equation $x^2 + y^2 = a^2, a > 0$

Polar equation $r = a, a > 0$

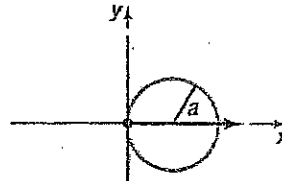
Typical graph



Passing through the pole, tangent to the line $\theta = \frac{\pi}{2}$, center on the polar axis, radius a

Rectangular equation $x^2 + y^2 = \pm 2ax, a > 0$

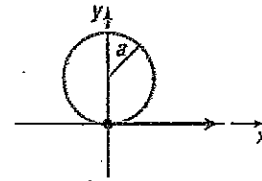
Polar equation $r = \pm 2a \cos \theta, a > 0$



Passing through the pole, tangent to the polar axis, center on the line $\theta = \frac{\pi}{2}$, radius a

Rectangular equation $x^2 + y^2 = \pm 2ay, a > 0$

Polar equation $r = \pm 2a \sin \theta, a > 0$

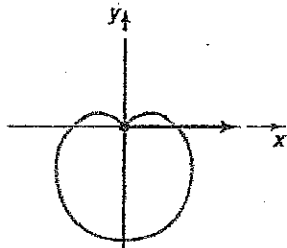


Other Equations

Name Cardioid

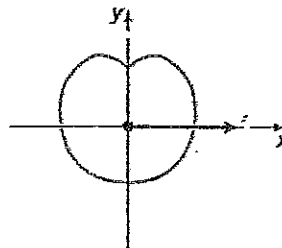
Polar equations $r = a \pm a \cos \theta, a > 0$
 $r = a \pm a \sin \theta, a > 0$

Typical graph



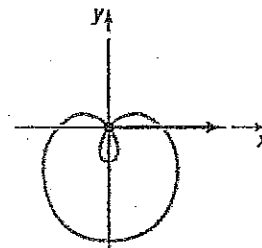
Limaçon without inner loop

$r = a \pm b \cos \theta, 0 < b < a$
 $r = a \pm b \sin \theta, 0 < b < a$



Limaçon with inner loop

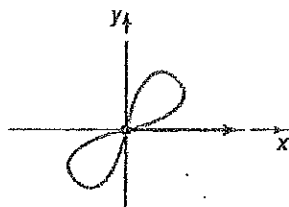
$r = a \pm b \cos \theta, 0 < a < b$
 $r = a \pm b \sin \theta, 0 < a < b$



Name Lemniscate

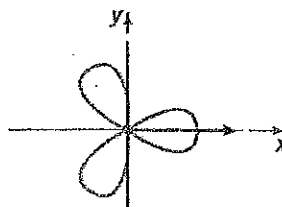
Polar equations $r^2 = a^2 \cos(2\theta), a > 0$
 $r^2 = a^2 \sin(2\theta), a > 0$

Typical graph



Rose with three petals

$r = a \sin(3\theta), a > 0$
 $r = a \cos(3\theta), a > 0$



Rose with four petals

$r = a \sin(2\theta), a > 0$
 $r = a \cos(2\theta), a > 0$

